This question paper contains 4 printed pages.

Your Roll No. .....

No. of Paper	: 753
Unique Paper Code	: 32357502
Name of the Paper	: **Mathematical Modelling and Graph Theory**
Name of the Course	: **B.Sc. (H) Mathematics : DSE-2**
Semester	: **V**
Duration	: **3 hours**
Maximum Marks	: **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt **all** *questions, selecting* **three** *parts from each question.*

(a) Solve the initial value problem using the Laplace transform: 6

$$x''-6x'+8x=2;\ x(0)=0,\ x'(0)=0.$$

(b) (i) Find the inverse Laplace transform of: 2

$$F(s)=\frac{5s-6}{s^2-3s}.$$

(ii) Show that: 2

$$L=\{t\sinh kt\}=\frac{2ks}{(s^2-k^2)^2}$$

(iii) Find the inverse Laplace transform of: 2

$$F(s)=\frac{1}{s^4-16}.$$

(c) Find two linearly independent Frobenius series solutions of: 6

P. T. O.

$$2x^2y''+xy'-(3-2x^2)y=0.$$

(d) Use power series to solve the initial va problem:

$$(x^2-6x+10)y''-4(x-3)y'+6y=0;\ y(3)=2, y'(3)=$$

2. (a) Explain Middle-Square Method and use it generate 10 random numbers taking $x_0=10$ Comment about the results. Was there cycli *Illustrate.*

(b) Using Monte Carlo Simulation, write an algori to calculate the area trapped between the curves $y=x^2$ and $y=6-x$ and the x- and axes.

(c) Using the simplex method:

Maximize $3x+y$

subject to $2x+y\le 6,$

$x+3y\le 9,$

$x,\ y\ge 0.$

(d) Using algebraic analysis:

Maximize $10x+35y$

subject to $4x+3y\le 24,$

$4x+y\le 20,$

$x,\ y\ge 0.$

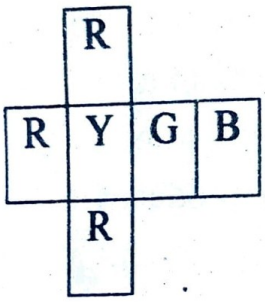
3. (a) (i) Determine the number of edges of $K_{9,10}$, Q_5 C_{10}.

(ii) Define complete bipartite graph. How many vertices and edges does a complete graph $K_{m,n}$ have? 3

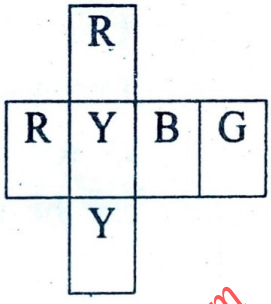
) (i) Prove that in any balanced signed graph every cycle has an even number of edges. 4

(ii) Draw a simple connected graph with degree sequence (3, 3, 3, 3, 3, 5, 5, 5). 2

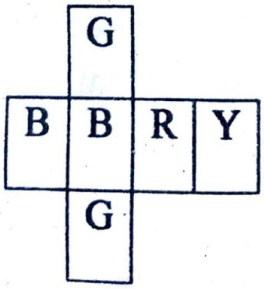
c) Determine whether the given four cubes having four colors, can be stacked in a manner so that each side of the stack formed will have all the four colors exactly once. 6



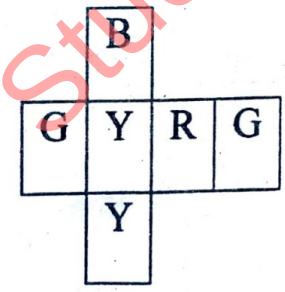
Cube 1



Cube 2



Cube 3



Cube 4

(d) Define a r-regular graph. Prove that, a r-regular graph with n vertices has $nr/2$ edges. 6

$$s^4 + 4a^4 = (s^2 - 2as + 2a^2)(s^2 + 2as + 2a^2)$$

and apply inverse Laplace transform to show

$$L^{-1}\left\{\frac{s^3}{s^4 + 4a^4}\right\} = \cosh at \cos at .$$

(b) Find the general solutions in power of x of following differential equation:

$$y'' + xy' + y = 0.$$

(c) A carpenter realizes a net unit profit of $ 2 table and $ 30 per bookcase. He has up to board-feet of lumber and up to 120 hours of to devote weekly to the project. The lumbe the labor can be used productively elsewh not used in the production of tables bookcases. He estimates that it requires 20 feet of lumber and 5 hours of labor to comp table and 30 board-feet of lumber and 4 ho labor to complete a bookcase. For mathematical model and use graphical anal determine how many of each piece of furnit should make each week to maximize his pro

(d) Prove that there is no knight's tour on chessboard.